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INST CA CENTER FOR THE STUDY OF NONLINEAR DYNAMICS
1983 LJI-R-83-262 N00014-79-C-0537

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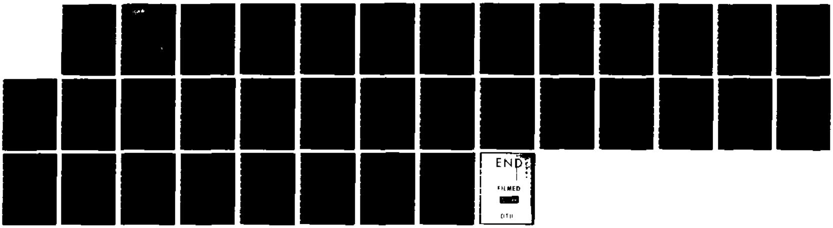

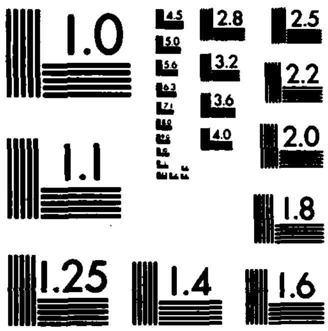
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LaJolla INSTITUTE
CENTER FOR STUDIES OF NONLINEAR DYNAMICS
8950 VILLA LA JOLLA DRIVE • SUITE 2150
LA JOLLA • CALIFORNIA 92037 • PHONE (619) 454-3581

Final Report for Contract:

Number N-00014-79-C-0537

Center for Studies of Nonlinear Dynamics

For

Office of Naval Research
800 North Quincy Street
Arlington, VA 22217

From

Center for Studies of Nonlinear Dynamics
La Jolla Institute
8950 Villa La Jolla Drive, Suite 2150
La Jolla, CA 92037

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The Center for Studies of Nonlinear Dynamics (CSND).

I. INTRODUCTION

The Center for Studies of Nonlinear Dynamics (CSND) was formed by the La Jolla Institute (LJI) in the early summer of 1979. In the fall of 1979 the Office of Naval Research granted CSND its first financial support [Contract No. N00014-79-C-0537]. In this report we will review the successes of CSND accomplished under the auspice of this contract.

The original concept of the Center for Studies of Nonlinear Dynamics was that of an organization where junior postdoctoral scientists would work with senior scientists visiting the Center and with staff and faculty of the University of California and the Scripps Institution of Oceanography (SIO). A Scientific Advisory Board was organized and its members are shown in Table 1. The postdoctoral staff of the Center consists of promising young people from the physical sciences and applied mathematics (cf Table 2). This concept has proven to be uniquely successful. In the four years of operation of the Center for Studies of Nonlinear Dynamics under the ONR contract 0537, its members have organized and supported over 70 seminars; initiated a joint seminar program with Scripps Institution of Oceanography; written 74 reports in research areas of interest to the Navy*; published over 55 articles in refereed journals*; organized and hosted 8 workshops on timely topics; published the proceedings of three of the workshops; published one book on nonlinear gravity waves; taught two courses in nonlinear dynamics at the University of California, San Diego; and have made uncounted contributions to scientific meetings and government panels. Much of this work was in the area of physical oceanography and geophysical hydrodynamics and was accomplished by a group of never more than several post doctorals, two senior CSND staff members along with the Directors, Kenneth M. Watson (1979-81), Kenneth M. Case (1981-82), and Stanley M. Flatté (1982-), and the Associate Director Bruce J. West (1980-) in conjunction with a procession of senior visitors (see Table 3). A detailed summary of the activities of CSND is presented in Section II, and its research accomplishments in Section III.

*See Tables 4 and 5 for a list of these publications.

Table 1

Scientists that have served, or are presently serving on the Senior Advisory Board of CSND are:

Luis W. Alvarez, Nobel Laureate, Physics Department, University of California, Berkeley.

Roger F. Dashen, The Institute for Advanced Study, Princeton, New Jersey.

Joseph B. Keller, Department of Mathematics, Stanford University.

Elliot W. Montroll, Institute for Physical Science and Technology, University of Maryland.

Walter H. Munk, Institute of Geophysics and Planetary Physics, Scripps Institution of Oceanography, University of California, San Diego.

Kenneth M. Watson, Marine Physical Laboratory, Scripps Institution of Oceanography, University of California, San Diego.

Table 2

The postdoctorals and other junior members who were (are*) members of CSND for at least one year are:

Michael Arthur

George Carnevale

* Dennis Creamer

Robert Littlejohn

Neil Pomphrey

* Yitzhak Rabin

* Stephen Reynolds

Alice Roos

Legese Senbutu

Venkita Seshadri

Michael Tabor

* John Weiss

Table 3

A partial list of senior visitors to CSND during the period of the contract:

Henry Abarbanel	Lawrence Berkeley Laboratory
Hassan Aref	Brown University
Nils Bas	University of Trondheim
Curtis Callan	Princeton University
Y.F. Chang	University of Nebraska
David Chudnovsky	Columbia University
Gregory Chudnovsky	Columbia University
D.H. Corliss	Marquette University
Roger Dashen	Institute for Advanced Studies
D. DuBois	Los Alamos National Laboratory
M. Feigenbaum	Los Alamos National Laboratory
Uriel Frisch	Observatoire de Nice
J. Gibbon	Imperial College, London
John Greene	GA Technologies
Michael Gregg	University of Washington, Seattle
J. Hammack	University of California, Berkeley
Robert H.G. Helleman	Twente University of Technology
J. Herring	NCAR
Greg Holloway	Institute of Ocean Sciences
Mark Kac	University of Southern California
L. Kadonof	University of Chicago
Y. Kodama	Bell Laboratories
Martin Kruskal	Princeton University
Joel Lebowitz	Rutgers University
Peter Lomdahl	Los Alamos Center for Nonlinear Studies
M. Longuet-Higgins	University of Cambridge
D. Meiron	California Institute of Technology
B. Mattes	University of Michigan
Michael Milder	Areté Associates

Duane Montgomery	University of Regina, Saskatchewan
Elliott Montroll	University of Maryland
P. Morrison	Princeton University
Theo Nonnenmacher	University of Ulm, West Germany
Irwin Oppenheim	Massachusetts Institute of Technology
I. Percival	Queen Mary College, London
D.H. Peregrine	University of Bristol
R.P. Potts	University of Adelaide
H. Rose	Los Alamos National Laboratory
J. Richardson	University of California, Santa Barbara
R. Savit	University of California, Santa Barbara
Gyorgy Targonski	University of Marburg
Nicolaas G. van Kampen	University of Utrecht
J. Vesecky	Stanford University
Henry Yuen	TRW Space Technology
G. Weiss	National Institute of Health
Fredrik Zachariasen	California Institute of Technology

II. ACTIVITIES OF CSND

In this section we provide a somewhat detailed overview of the research conducted at CSND as well as some overall measures of the success of the research effort. Quantitative measures of the quality of research conducted at an institution are difficult to formulate. Universities have traditionally used such guides as the number of research publications and the Citation Index to determine the quality of individual research efforts for faculty promotions. In addition such considerations as teaching, involvement in academic affairs and personal recommendations and evaluations have been used. We can apply many of these same measures to the research accomplishments of the Center.

In Table 4 we give a cumulative list of publications by the members of the Center in refereed journals. Note that the 65 articles relate only to those activities supported whole or in part by the Office of Naval Research contract. The overall level of activity of the Center is at least twice this value. In Table 5 is given a cumulative list of unpublished reports (19) which differ from those in Table 4. These reports very often were in direct response to questions asked by the sponsor and therefore were not appropriate to publish in the more general context of a scientific journal. The final indicator of published work is given in Table 6 where we have listed the books authored and/or edited by members of CSND.

As a direct consequence of the quality of research conducted under this contract the University of California, San Diego (UCSD) and the La Jolla Institute have entered into an agreement to affiliate the Institute's Center for the Studies of Nonlinear Dynamics (CSND) with UCSD. The agreement was signed on May 31, 1983 by UCSD Chancellor Richard C. Atkinson, Dr. Adolf R. Hochstim, president of the La Jolla Institute, and Dr. Stanley M. Flatté, director of CSND and professor of physics currently on leave from UC, Santa Cruz. The purpose of the agreement is to facilitate greater cooperation and exchange of information between UCSD and the Center.

The following is a direct quotation from the newspaper *La Jolla Light*, June 9, 1983:

> The field of nonlinear dynamics is the study of physical and natural events which do not behave in an easily predictable manner. Weather patterns, the behavior of ocean waves, and fluctuating chemical processes are examples of nonlinear dynamic problems currently being studies at CSND.
>
> "We welcome this opportunity to strengthen our ties with this outstanding research facility," Atkinson said of the new agreement, "I have no doubt that both the campus and the Center will benefit greatly from the improved sharing of ideas which will ultimately stem from this relationship."
>
> The new agreement will allow UCSD graduate students to pursue their research at CSND and permit scientists from the Center to serve on their dissertation committees.
>
> "An important contribution of the La Jolla Institute to research activities of CSND is the organization and hosting of workshops and conferences of international scope," Flatté said.
>
> Under terms of the agreement, scientists at the Center will be able to use university facilities such as the computer center and libraries with the same degree of access as regular UCSD faculty. Center scientists will also be given research titles at the university appropriate to their professional standing.

The affiliation of CSND with UCSD stands as testimony to the involvement of the scientists at the Center with the academic affairs of the University.

The final measure we mention is the response of the scientific community to the conferences and workshops organized and hosed by the Center. In Table 7 the eight workshops so organized are listed. The first three workshops were intended primarily for local scientists such as those the Scripps Institution of Oceanography. Of the latter five, two were intended primarily for scientists within the United States (*Nonlinear Properties of Internal Waves* and *Mathematical Methods in Hydrodynamics and Integrability in Dynamical Systems* both published by the American Institute of Physics) and the remaining three were truly international in scope. *The Predictability of Fluid Motion*, also published by the American Institute of Physics, had participants from England, France, Germany (F.R.), Canada, Italy, Poland, Mexico, Japan, and China. *Dynamics Days*, the first and second annual workshop on nonlinear dynamics, attracted participants from these countries and others and has been one of the principle world-wide meetings for the nonlinear dynamics community. The purpose of the latter workshops is to provide an overview of the most

recent developments in: chaotic behavior, mappings of the interval and plane, classical mechanics, turbulence, quantum aspects of chaotic behavior, solitons, Arnold diffusion, invariant tori, strange attractors, etc. Thus on the order of seventy-five (75) talks are given in a three day period by the leading experts in the exploding field of nonlinear dynamics. This workshop provides a unique opportunity for the members of CSND to exchange ideas and formulate future research plans with a broad spectrum of scientists.

The major research areas in which the members of CSND have been engaged are: 1) hydrodynamics; 2) physical oceanography; 3) non-equilibrium statistical mechanics and 4) nonlinear dynamics. Although there is a certain amount of overlap in these categories, they do serve to indicate the relative stress of a given avenue of research. The research which has been completed under the ONR contract, is reviewed in the next section.

Table 4

Journal publications by members of CSND under contract N-00014-79-0527.

1980

1. Henyey, Frank S., Improved Ray Description of Wave Equations, Phys. Rev. Lett. **45**, 1897-1900 (1980).

2. Lindenberg, Katja, Seshadri, V., and West, Bruce J., Brownian Motion of Harmonic Systems with Fluctuating Parameters, II. Relation Between Moment Instabilities and Parametric Resonance, Phys. Rev. A **22**, 2171 (1980).

3. Lindenberg, Katja, Seshadri, V., Shuler, K. E., and West, Bruce J., Equal-Time Second-Order Moments of a Harmonic Oscillator with Stochastic Frequency and Driving Force, J. of Stat. Phys., **23**, 755 (1980).

4. Seshadri, V., West, Bruce J., and Lindenberg, K., Analytic Theory of Extreme, II: Application to Nonlinear Oscillators, J. Sound Vib., **68** (4), 553-570 (1980).

5. Seshadri, V., West, Bruce J., Lindenberg, Katja, Analytic Theory of Extreme III. Results for Master Equations with Application of Unimolecular Decomposition, J. Chem. Phys., **72**, 1145 (1980).

6. West, Bruce J., Lindenberg, Katja, and Seshadri, V., Analytic Theory of Extrema. IV. Relaxation from Metestable and Unstable States, J. Chem. Phys., **72**, 1151-1155 (1980).

7. West, Bruce J., Lindenberg, Katja, and Seshadri, V., Brownian Motion of Harmonic Systems with Fluctuating Parameters, I. Exact First and Second Order Statistics of a Mechanical Oscillator, Physica A, **102A**, 470-488 (1980).

1981

8. Carnevale, G. F., Statistical Field Theory and the Internal Wave Problem in NONLINEAR PROPERTIES OF INTERNAL WAVES, ed. Bruce J. West, La Jolla Institute Workshop, AIP Conf. Proc. **76** (1981).

9. Case, K. M., Dual Hamiltonian Formulations and Completely Integrble Systems, in MATHEMATICAL METHODS IN HYDRODYNAMICS AND INTEGRABILITY IN DYNAMICAL SYSTEMS, eds. M. Tabor and Y. M. Treve, La Jolla Institute Workshop, AIP Conf. Proc. **88**, 163 (1981).

10. Chang, Y. F., Tabor, M., Weiss, J., and Corliss, G., On the Analytic Structure of the Henon-Heiles System, Phys. Lett. **85A**, 211 (1981).

11. Henyey, F. S., Gauge Groups and Noethers' Theorem for Continuum Mechanics, in MATHEMATICAL METHODS IN HYDRODYNAMICS AND INTEGRABILITY IN DYNAMICAL SYSTEMS, La Jolla Institute Workshop, AIP Conf. Proc. **88** (1981).

12. Henyey, F. S., Comment on the Use of Eikonal Techniques for Induced Diffusion in NONLINEAR PROPERTIES OF INTERNAL WAVES, ed., West, Bruce J., La Jolla Institute Workshop, AIP Conf. Proc. **76**, 339-343 (1981).

13. Lindenberg, Katja and Seshadri, V., Dissipative Contributions of Internal Multiplicative Noise. I. Mechanical Oscillator, Physica **109A**, 483 (1981).

14. Lindenberg, K., Seshadri, V., and West. B. J., Brownian Motion of Harmonic Systems with Fluctuating Parameters III. Scaling and Moment Instabilities, Physica **105A**, 219 (1981).

15. Manley, O. P. and Treve, Y. M., Minimum Number of Modes in Approximate Solutions to Equations of Hydrodynamics, Phys. Lett. **82A** No. 2 (1981).

16. Meiss, James D. and Watson, K. M., Relaxation Processes for a Three- Wave Interaction Model, Proc. Natl. Acad. Sci., USA, **78**, No. 4, 2029-2032 (1981).

17. Olbers, Dirk J. and Pomphrey, Neil, Disqualifying Some Candidates for the Energy Balance of Oceanic Internal Waves, J. Phys. Oceanogr. **11**, 1423-1425 (1981),

18. Pomphrey, N., Review of Some Calculations of Energy Transport in a Garrett-Munk Ocean, in NONLINEAR PROPERTIES OF INTERNAL WAVES, ed., Bruce J. West, AIP Conf. Proc. **76** (1981).

19. Seshadri, V. and Lindenberg, Katja, Dissipative Contributions of Internal Multiplicative Noise, II. Spin Systems, Physica **115A**, 501 (1981).

20. Seshadri, V., West, Bruce J., Lindenberg, Katja, Stability Properties in Nonlinear Systems with Fluctuating Parameters, I. Nonlinear Oscillators, Physica **107A**, 219-240 (1981).

21. Tabor, Michael and Weiss, J., Analytic Structure of the Lorenz System, Phys. Rev. **A24**, 2157 (1981).

22. Tabor, Michael, Regular and Chaotic Regimes in Quantum Mechanics, in QUANTUM MECHANICS IN MATHEMATICS, CHEMISTRY AND PHYSICS, eds., U. E. Gustavson and W. P. Reinhardt, Plenum, New York (1981).

23. Tabor, Michael, The Nonlinear Dynamics of the Classical Few Body Problem, Nuc. Phys. **A353**, 353c (1981).

24. Treve, Y. M., A Numerical Test of the Reliability of Galerkin Approximations to the Solutions of the Navier-Stokes Equation, J. Comp. Phys. **41**, 217 (1981).

25. Treve, Y. M., A Numerical Test of the Reliability of Galerkin Approximations to the Solutions of the Navier-Stokes Equation, J. Comp. Phys. **40**, No. 2 (1981).

26. Watson, K. M., Internal Wave Transport at High Vertical Wavenumbers: The Elastic Scattering Mechanism in NONLINEAR PROPERTIES OF INTERNAL WAVES, ed. Bruce J. West, AIP Conf. Proc. **76** (1981).

27. West, Bruce J. and Seshadri, V., Model of Gravity Wave Growth Due to Fluctuations in the Air-Sea Coupling Parameter, J. Geophy. Res., **86**, No. C5, 4293 (1981).

28. West, Bruce J., Steady State Spectral Density of Gravity-Capillary Waves, J. Geophys. Res. **86**, C11, 11073 (1981).

29. West, Bruce J., Non-Gaussian Water Waves, in MATHEMATICAL METHODS OF HYDRODYNAMICS AND INTEGRABILITY IN RELATED DYNAMICAL SYSTEMS, AIP Conf. Proc. **88**, 347, eds. M. Tabor and Y.M. Treve (1981).

30. Aref, Hassan and Pomphrey, Neil, Integrable and Chaotic Motions of Four Vortices, Phys. Lett. **78A**, 297 (1980).

1982

31. Carnevale, G. F. and Holloway, G., Information Decay and the Predictability of Turbulent Flows, J. Fluid Mech. **116**, 115-121 (1982).

32. Carnevale, G. F., Statistical Features of the Evolution of Two- Dimensional Turbulence, J. Fluid Mech. **122**, 143-153 (1982).

33. Carnevale, G. F., A Nonstationary Solution to Liouville's Equation for a Randomly Forced Two-Dimensional Flow with Rayleigh Friction, Phys. Fluids **25**, 1527-1549 (1982).

34. Case, K. M. and Roos, A., Sine-Gorcon and Modified Korteweg de Vries Charges, J. Math. Phys. **23**, No. 3, 392 (1982).

35. Case, K. M. and Arthur, M. D., Gradient Theorem for Completely Integrable Hamiltonian Systems, J. Math Phys. **10**, 1771 (1982).

36. Chang, Y. F., Tabor, M., and Weiss, J., Analytic Structure of the Helon-Heiles Hamiltonian in Integrable and Nonintegrable Regimes, J. Math. Phys. **23**.

37. Henyey, F. S. and Seshadri, V., On the Number of Distinct Sites Visited in 2-D Lattices, J. Chem. Phys. **76**, 5503 (1982).

38. Henyey, F.S., Canonical Construction of a Hamiltonian for Dissipation-Free MHD, Phys. Rev. A **26**, 480 (1982).

39. Henyey, F. S. and Pomphrey, N., The Autocorrelation Function of a Pseudointegrable System, Physica **6D**, 78-94 (1982).

40. Henyey, F.S., The Distinction Between a Perfect Conductor and a Superconductor, Phys. Rev. Lett **49**, 416 (1982).

41. Henyey, F. S. and Pomphrey, N., Self-Consistent Elastic Moduli of a Cracked Solid, Geophys. Res. Lett **9**, 903-906 (1982).

42. Lindenberg, Katja, Seshadri, V., and West, Bruce J., Dynamical Behavior of Model Oscillators with Fluctuating Frequency and Driving Force, in INSTABILITY BIFURCATIONS, AND FLUCTUATIONS IN CHEMICAL SYSTEMS, eds. L. E. Reichl and W. C. Schieve, U. of Texas Press, Austin, 411 (1982).

43. Meiss, J. D., and Watson, K. M., Internal Wave Interactions in the Induced Diffusion Approximation, J. Fluid Mech. **117**, 315 (1982).

44. Montgomery, W. Duane, Optical Applications of von Neumann's Alternating Projection Theorem, Optics Lett. **7**, No. 1, 1-3 (1982).

45. Montgomery, W. Duane, Restoration of Images Possessing a Finite Fourier Series, Optics Lett. **7**, No. 2, 54-56 (1982).

46. Montgomery, W. Duane, Diffraction by Perfectly Conducting Plane Screens II, Il Nuovo Cimento **71**, 65-74 (1982).

47. Seshadri, V. and West, Bruce J., Fractal Dimensionality of Lévy Processes, Proc. Nat. Acad. Sci. USA,**79**, 4501, (1982).

48. Seshadri, V., Lindenberg, K., and West, B. J., Theory of Extrema for Fokker-Planck and Master Equation Processes, in INSTABILITIES, BIFURCATIONS, AND FLUCTUATIONS IN CHEMICAL SYSTEMS, eds. L. E. Reichl and W.C. Schieve, University of Texas Press, Austin, 383 (1982).

49. Tabor, M., Integrability and Analytic Structure of Dynamical Systems, Int. J. Quant. Chem., **S16**, 167 (1982).

50. Weiss, J., Analytic Structure of the Henon-Heiles System in MATHEMATICAL METHODS IN HYDRODYNAMICS AND INTEGRABILITY IN DYNAMICAL SYSTEMS, eds. M. Tabor and Y. M. Treve, AIP Conf. Proc. **88** (1982).

51. West, Bruce J., Resonant Test Field Model of Fluctuating Nonlinear Wave, Phys. Rev. **A25**, (1982).

52. West, Bruce J., Stochastic Predictions of Cataclysmic Events, in INTERPRETATION OF CLIMATE AND PHOTOCHEMICAL MODELS, OZONE AND TEMPERATURE MEASUREMENTS, eds. Reck, R. and Hummel, J., La Jolla Institute Workshop, AIP Conf. Proc. **82**, 263 (1982).

53. West, Bruce J. and Seshadri, V., Linear Systems with Lévy Fluctuations, Physica, *113A*, 203 (1982).

54. West, Bruce J., Statistical Properties of Water Waves, I. Steady State Distribution of Wind Driven Gravity-Capillary Waves, J. Fluid Mech. *177*, 187 (1982).

1983

55. Carnevale, G. F. and Fredericksen, J., Viscosity Renormalization Based on Direct Interaction, J. Fluid Mech. (1983).

56. Carnevale, G. F. and Martin, P. C., Field Theoretical Techniques in Statistical Fluid Dynamics: With Application to Nonlinear Wave Dynamics, Geophys. Astrophys. Fluid Dyn. *20*, 131-164 (1982).

57. Chang, Y. F., Greene, J. M., Tabor, M., and Weiss, J., Analytic Structure of Dynamical Systems and Self-Similar Natural Boundaries, to be published in Physica D (1983).

58. Greene, J. M., Some Order in the Chaotic Regimes of Two-Dimensional Maps, in LONG-TIME PREDICTION IN DYNAMICS, ed. C. W.. Horton, Jr., L. E. Reichl, and A. G. Szebehely, John Wiley & Sons, Inc. (1983).

59. Henyey, F. S., Hamiltonian Description of Stratified Fluid Dynamics, Phys. of Fluids, **26**, 40 (1983).

60. Henyey, F. S. and Pomphrey, N., Eikonal Description of Internal Wave Interactions: A Non-Diffusive Picture of "Induced Diffusion", Dyn. Atmos. and Oceans (to appear) (1983).

61. Lindenberg, Katja, Shuler, Kurt E., Seshadri, V., and West, Bruce J., Langevin Equations with Multilicative Noise. Theory and Applications to Physical Processes, to appear in Probabilistic Analysis and Related Topics, ed. A. T. Bharucha-Reid, Academic Press (1983).

62. Montgomery, W. Duane, Phase Ambiguities in Higher Dimensions, Nuovo Cimento (to appear) (1983).

63. West, Bruce J., Statistical Properties of Water Waves, IV. Exact Equilibrium Properties of RTF, La Jolla Institute Report, LJI-TN-81-130, to be published in the Proceedings from the ICURM Conference in Miami (1983).

64. West, Bruce J., A Resonant Test Field Model of Deep Water Gravity Waves, J. Fluid Mech. *132*, 417 (1983).

65. Wright, Jon A. and Dewitt, R.J. Self-consistent Effective Medium Parameters for Oceanic Internal Waves, in J. Fluid Mech. (to appear) (1983).

Table 5

CSND unpublished reports under contract N-00014-79-0537 (distinct from those in Table 4).

1980

66 "On a Statistical Model of Two-Dimensional Turbulence" Bruce J. West, LJI-TN-79-046.

67 "On the Fourmulation of Methods of Artificial Compressibility for the Navier-Stokes Equation" John Weiss, LJI-TN-80-069.

68 "Status Report on the Theory of Electromagnetic Internal Wave Measurements" Frank Henyey, LJI-TN-80-072.

69 The Onset of Turbulence and Strange Attractors" M. Tabor, LJI-TN-80-073.

70 "Statistical Properties of Water Waves. II. A Resonant Test Field Model of Gravity Waves" Bruce J. West, LJI-R-80-074.

71 "Notes on the Steady State Properties of Deep Water Gravity Waves" Bruce J. West, LJI-TN-80-079.

72 "Note Concerning the Propagation of a Single Internal Wave" Kenneth M. Watson, LJI-TN-80-088.

73 "Weakly Nonlinear Solutions of Long's Equations" Frank S. Henyey, LJI-TN-80-095.

1981

74 "Hamiltonian Description of Internal Waves and Dynamics" Frank S. Henyey, LJI-TN-81-109.

75 "Statistical Properties of Water Waves III Nonlinear Equilibrium Properties of Resonant Test Field" Bruce J. West, LJI-R-81-114.

76 "The Dynamics of Enstrophy Transfer in Two Dimensional Hydrodynamics" J. Weiss, LJI-Tn-81-121.

77 "Hamiltonian Perturbation Theory in Noncanonical Coordinates" R.G. Littlejohn, LJI-TN-81-123.

78 "On Explicit, Conservative Difference Scheme for Hamiltonian Mechanics" John Weiss, LJI-TN-81-131.

79 "On the Calculation of Negative and Non-Integer Moments" G.H. Weiss and Bruce J. West, LJI-R-81-154.

80 "Review of the Spectral Evolution of Wind Generated Water Waves" Bruce J. West, LJI-R-81-157.

1982

81 "Poission Brackets for Scattering Data" K.M. Case, LJI-R-82-177.

82 "Surface Enhanced Raman Scattering" L. Senbetu, LJI-R-82-182.

83 "Canonical (Feynman Diagram) versus Non Canonical (Stokes Expansion) Calculation of Resonant Interaction Between Surface Waves" F.S. Henyey and N. Pomphrey, LJI-R-82-183

84 "Radiation by a Charged Particle Acceleration a Limited Region of Space" L. Senbetu and Bruce J. West.

Table 6

Books authored and/or edited by members of CSND under contract N-00014-79-C-00537.

85 West, Bruce J., ON THE SIMPLER ASPECTS OF NONLINEAR FLUCTUATING DEEP WATER GRAVITY WAVES (Weak Interaction Theory), Lecture Notes in Physics, Series **146**, Springer-Verlag, Berlin (1981).

86 Tabor, M. and Treve, Y.M., editors, MATHEMATICAL METHODS IN HYDRODYNAMICS AND INTEGRABILITY IN DYNAMICAL SYSTEMS, La Jolla Institute Workshop, AIP Conf. Proc. **88**, (1981).

87 West, Bruce J., editor, NONLINEAR PROPERTIES OF INTERNAL WAVES, La Jolla Institute Workshop, AIP Conf. Proc. **76**, (1981).

88 Holloway, G. and West, Bruce J., editors, PREDICTABILITY OF FLUID MOTION, La Jolla Institute Workshop, to appear as AIP Conf. Proc. (1983).

Table 7

Workshops organized by the Center fo Studies of Nonlinear Dynamics, supported whole or in part by Contract Number N-00014-79-C-00537.

14-18 SEPTEMBER 1979
Efficient Modal Expansions
Organized by: Kenneth Watson, CSND
Location: Scripps Institution of Oceanography

19-21 SEPTEMBER 1979
Coherent Structures in Turbulent Laboratory and Geophysical Flows
Organized by: Myrl Hendershott, Scripps Institution of Oceanography
Location: Scripps Institution of Oceanography

15-17 OCTOBER 1980
Nonlinear Feeback Mechanism in Natural Phenomena
Organized by: Kenneth Watson, CSND, and Elliott Montroll, University of Rochester
Location: Scripps Institution of Oceanography

15-17 JANUARY 1981
Nonlinear Properties of Internal Waves
AIP Conf. Proc., **76**, 1981
Organized by: Kenneth Watson and Bruce J. West, CSND
Location: Scripps Institution of Oceanography

7-9 DECEMBER 1981
Mathematical Methods in Hydrodynamics and Integrability in Dynamical Systems
AIP Conf. Proc., **88**, 1982
Organized by: Michael Tabor and Yvain Treve, CSND
Location: Scripps Institution of Oceanography

5-7 JANUARY 1982
Dynamics Days, First Annual Workshop on Nonlinear Dynamics
Organized by: Robert H.G. Helleman, Twente University and Charles Eminihizer; Physical Dynamics, Inc.
Location: La Jolla Village Inn, La Jolla

4-6 JANUARY 1983
Dynamics Days, Second Annual Workshop on Nonlinear Dynamics
Organized by: Robert H.G. Helleman, Twente University and Charles Eminihizer, Physical Dynamics, Inc.
Location: Sheraton-Airport Inn, San Diego

1-4 FEBRUARY 1983
The Predictability of Fluid Motion
Organized by: Bruce J. West, CSND and Greg Holloway, Institute of Ocean Sciences
Location: Scripps Institution of Oceanography

III. DETAILS OF RESEARCH CONDUCTED AT CSND

A. HYDRODYNAMICS

The traditional methods of hydrodynamics are well established and are used in a great many applications with confidence. There are, however, a number of research areas in this category which are on the periphery of our present understanding. Phenomena such as turbulence have long eluded systematic study and the analytic structures of hydrodynamic-like equations have been uncertain. To examine such questions certain members of CSND have undertaken research into the development of numerical techniques which may reduce the intrinsic difficulty in solving partial differential equations. This work will be discussed in Section D. Other members of the Center have taken a different approach. These latter approaches are described in this section.

ACCOMPLISHMENTS†

A.1 Integrable Systems

Until recently most problems of physical interest that could be solved analytically were linear. However, any view of an ocean clearly shows that the nonlinear effects are important. In the past 20 years it has been found that there are classes of nonlinear equations which are completely integrable. Even more remarkably it has turned out that many of these are applicable to interesting physical situations. For example, the Korteweg-deVries, Benjamin-Ono, and the Nonlinear Schrödinger equations all describe (rather well) solutions of the equations of hydrodynamics in particular regimes.

The point of the research done in this area is to analyze common properties of the completely integrable systems mentioned above and to find new properties. Thus in the work by K. Case and A. Roos[34] it is shown that a number of equations of physical interest are very closely related. Indeed they have the

†The numbering of references in this and later sections refers to the publications list in Tables 4 and 5

same constants of motion. In addition, it is shown that a whole new infinite class of constants exists.

The study by K. Case points out that almost all the simple properties of known completely integrable systems follows from the fact that the systems can be written in Hamiltonian form in two ways[9]. Thus the existence and construction of an infinite set of constants and the fact that these constants are in involution is immediate. Construction of operators such that the Inverse Scattering Transform Method can be used is also a consequence of the dual Hamiltonian nature of the system.

The method implies a recursion relation for certain functions. For the first few of these functions one can readily verify that these functions are gradients of conserved functionals. A general proof is given by Case and Arthur[35]. The results of Case and Arthur are used by Case[61] to compute Poisson Brackets between the scattering data of a large class of evolution equations.

A.2 Nonequilibrium Statistical Mechanics

Langevin equations for closed systems with multiplicative fluctuations must also include dissipative terms that ensure eventual equilibrium of the system. Lindenberg and Seshadri[13] consider an oscillator coupled to a heat bath and show that a particular nonlinear coupling to a harmonic heat bath leads to a fluctuating frequency and to nonlinear dissipative terms. They also analyze the effects of the multiplicative fluctuations and of the corresponding nonlinear dissipation on the temporal evolution of the average oscillator energy. They find that the rate of equilibration of this system can be significantly different from that of an oscillator with only additive fluctuations and linear dissipation.

The analysis in this paper has been applied to a field of nonlinear deep water gravity waves[51,64] and is discussed in Subsection B.3 and C.1.

B. PHYSICAL OCEANOGRAPHY

Under this heading we have gathered together those pieces of research which have direct application to particular physical mechanisms of importance in ocean physics. In particular, we have applied certain theoretical models (to be described in Section C) to the generation, evolution, and eventual relaxation of wind generated water waves on the ocean surface into a steady state spectrum. Such techniques have also been employed in the description of the internal wave spectrum.

ACCOMPLISHMENTS

B.1 Nonlinear Properties of Internal Waves (workshop)[87]

A variety of observed data have suggested a "universal equilibrium" spectrum for internal wave within the deep ocean (Garrett and Munk, referred to as GM). It is thought that this spectrum is maintained by energy sources balanced by energy sinks and that energy is transported in wavenumber space through the spectrum principally be nonlinear wave-wave interactions. A workshop was hosted by CSND at SIO focusing on the present level of understanding of the nonlinear properties of internal waves[87]. The consensus of the workshop is that a flow of energy from the high frequency-low horizontal wavenumber to the low frequency-high horizontal wavenumber internal waves is maintained [cf. (Figure 1)].

The unshaded domain of Figure 1 is dominated by the mechanism labeled *induced diffusion* (ID). The induced diffusion mechanism describes the interaction of relatively small scale, higher frequency waves with larger scale waves of frequency near the inertial frequency. The ID-interactions cause energy to flow to the right side of the diagram. Along the boundary labeled (e) the waves are thought to be unstable and to break.

As was observed by Holloway† and also by Pomphrey, Meiss and Watson the

†See Reference 87 for all references used in this discussion.

interactions in the induced diffusion regime and to the right of the dashed curve of Figure 1 is too strong for the weak interaction models on which the calculations were based to be valid. Meiss and Watson[16] noted that the induced diffusion mechanism could be described by the Taylor-Goldstein equation. This equation itself describes the propagation of internal waves through a vertically sheared large scale flow. Much of the analysis presented at the workshop supported the use of the Taylor-Goldstein equation as a reasonable descriptor of the energy transport process in the presence of shear flow[87].

Additional detailed discussions of the physical mechanisms responsible for the direction of energy flow in the "equilibrium" internal wave spectrum can be found in the proceedings.

B.2 Eikonal Approach to Nonlinear Properties Internal Wave[60]

Frank Henyey and Neil Pomphrey have completed a numerical study of the "induced diffusion" kinematic regime of internal wave interactions, using an eikonal representation. In this study, the small scale internal wave field is represented as a superposition of many small wavepackets, and the phase space (physical and wavenumber space) trajectories of typical packets are followed. The numerical results led them to propose a qualitative picture of the transport properties of the small waves in which the transport is dominated by "critical layer" events, i.e., a behavior of the wave packet very similar to the approach to a critical layer in a shear flow that is independent of time and horizontal position.

Their picture is very much at odds with the induced diffusion model coming from weak interaction transport theory. In a follow-up study done since the end of the period of this contract, they have been able to establish that the strength of internal wave interaction in the kinematic regime being considered is indeed too large for the validity of induced diffusion. As a result all the standard consequences of the weak interaction assumption (such as gaussian statistics for the wave amplitude) are likely to be far from the truth.

This work has led to a current project (supported by ONR) in which we investigate the consequences of the more intricate statistical properties of small scale internal waves to the breaking of the internal waves and the formation of microstructure. The computer codes developed during the contract period are essential in this present work.

The knowledge gained from this study has important implications for other oceanic processes with scale separation, notably in the interaction between surface waves and internal waves.

B.3 Review of Wind Generated Surface Wave Models[80]

The complex turbulent structure of the ocean waves has led to a variety of statistical descriptions of their properties. In particular, the fetch dependent wavenumber spectrum $\Phi(\boldsymbol{k}, \boldsymbol{x}, t)$ has provided a useful representation of ocean waves. To describe the evolution of this spectrum a number of investigators have introduced transport equations similar to that of radiation transfer or to the Boltzman equation in kinetic theory. Such transport equations express the rate of change of the spectrum as a sum of terms which individually model physical phenomena thought to be important in the development of ocean waves. Of particular significance are the wind generation of waves, the nonlinear wave-wave interactions, the effects of turbulent and molecular dissipation, the effect of imposed surface and drift currents and the effect of wave breaking. There is however a controversy over how one implements the understanding of these phenomena in the development of a practical sea state prediction code. In particular, the the JONSWAP group replaces the above transport approach by a parametric description of the evolution process assuming a self-similar spectral form. The evolution of the *parameters* in the self-similar spectrum are then calculated numerically.

A review of the present state of our understanding of the evolution of the wind generated wave field on the ocean surface has been completed[80]. The topics that are considered in this review are: 1) invicid boundary layer theory; 2) alternative linear instability mechanisms including the effects of viscosity and

the modulation of wind field fluctuations by the surface; 3) the general linear transport equations (validations by data); 4) nonlinear wave-wave interactions and how they modify the general transport equation for an evolving field of waves; 5) phenomenological models of wave breaking; 6) how these nonlinear effects are modeled in global scale numerical codes; 7) the parametric prediction models for the wave field, and 8) the scaling of laboratory data to the open ocean.

The conclusions of this study are as follows:

1. There is to date insufficient published comparisons between transport predictions and observational data to determine which of the above model most adequately represents the evolution of wind wave spectra on the ocean surface†. However, it appears that even the best is probably not adequate.

2. Only the spectral transport models have a scaffolding of theoretical support even though the expansions of the Hamiltonian on which they are based have not been shown to be convergent. However, even this limited theoretical support is eroded by phenomenology in the production codes such as SOWM. This erosion occurs when effects such as the wave-wave interactions are *replaced in total* by arbitrary limiting functions such as discussed in Section 3 of Reference 80.

3. The majority of both theoretical and experimental studies have concluded that the nonlinear interaction among gravity waves is the dominant physical mechanism for energy transfer from one spectral region to another. It, therefore, seems that greater research effort is warranted in both developing efficient calculational algorithms for the wave-wave interaction terms in the spectral transport equation and in constructing justifiable approximate forms for these nonlinear terms.

4. The nonlinear interactions among deep water waves have usually, in the past twenty years, been discussed in terms of resonant and non-resonant interactions. Resonant wave-wave interactions provide a slow energy transfer across the spectrum, whereas the non-resonant interactions give use to a periodic energy interchange among waves but yield no *net* energy transfer. Recent field experiments indicate that the non-resonant interactions are much more important than thought previously. Thus, the relative importance of these two interactions in the wave evolution process must be established in order to properly interpret remote sensing data.

5. Most of the processing techniques applied to data from the remote sensing of the sea surface rely on assumptions about the

†A study which has just been concluded (but is yet unpublished) may rectify this situation. (T. Barnett, private communications).

statistical properties of the surface waves. One usually argues that because the nonlinear interaction is weak that the water wave field is a Gaussian random field. However, such phenomena as wave "groupiness," wave breaking and the spectral width of the wave field would all argue against such an assumption. It, therefore appears that the influence of non-Gaussian statistics of the water wave field on remote sensing data should be systematically investigated.

6. The statistical nature of the water wave field is at least as important as the deterministic wave dynamics. This is apparent in the early work of Lonquet-Higgins (see West[85] for a review) in which the steady state statistics of water waves was examined. However, the evolution of the statistical properties of the wind generated water waves has not been treated with the same degree of rigor as the dynamics. The change in the statistics of the wave field as the energy spectrum evolves as a function of fetch is one of the limiting factors in the energy transfer process. West[54,64] has examined this process using a particular model and has been able to calculate a steady state spectrum for the water wave field as the limit of the dynamic process. This is the first calculation of a water wave spectrum based on a dynamic model, not a scaling argument. As such, it indicates that further studies of the statistics of non-equilibrium, non-linear wave fields could be of substantial assistance in understanding the evolution of wind generated deep water gravity waves.

B.4 Statistical Properties of Water Waves

In a series of reports the steady state water spectrum generated by a turbulent wind blowing over a water surface is investigated[28,54,64].

The Miles-Phillips model of the linear coupling between waves on the ocean surface and a fluctuating wind field is generalized to include the average effect of the nonlinear water-wave interactions in the dynamic equations for gravity-capillary waves. A statistical-linearization procedure is applied to the general problem and yields the optimum linear description of the nonlinear terms by linear terms[28,54]. The linearized dynamic equations are stochastic with solutions that have stable moments, i.e. the average nonlinear interactions quench the linear instability generated by the coupling to the mean wind field. In particular, an asymptotic steady-state power-spectral density for the water-wave field is calculated exactly in the context of the model for various wind speeds.

The mode rate equation derived from a Hamiltonian for the deep water gravity wave field separates the nonlinear interactions into resonant and non-

resonant terms[64]. The non-resonant interactions induce rapid phase variations and are modeled as a "heat bath" in which a resonant test field (RTF) of gravity waves evolves. The RTF models the slow resonant energy transfer process by a master equation for the phase space probability density. An H-theorem for the RTF predicts an asymptotic steady state near which the transport properties of the field are studied. The steady state energy spectral density obtained is the observed k^{-4}.

The water wave generation model of West and Seshadri[27] was generalized to include the average effect of the nonlinear interactions among the waves. In summary it is observed that this linearized model of the surface wave dynamics describes the asymptotic statistical steady state as a non-Gaussian wave field. The coherent pressure fluctuations give rise to a hyperbolic probability density in the action and results in a clustering of the waves on the sea surface, i.e., the waves appear in groups[29]. In addition the model predicts a steady state frequency spectrum which is not unreasonable.

B.5 Disqualifying Some Candidates for the Energy Balance of Oceanic Internal Waves[17]

Bell (1975) has suggested that internal tides generated by interaction of the barotropic tidal current with abyssal topography will spread their energy over the internal wave spectrum by weakly nonlinear coupling of the tidal wave components with waves in the internal wave continuum. We have extended the numerical work of Pomphrey, Meiss and Watson (1980) to calculate Langevin decay rates for internal waves in the low-frequency, low-modenumber domain. Our results show that the spreading of tidal energy is not an effective candidate for the overall spectral balance of the internal wave field. Similarly, we have given a crude estimate of the size of the source term that governs spectral transfer by scattering of waves at irregularities of the ocean flow, and concluded that, compared to the redistribution of energy by wave-wave interactions, the bottom scattering in unimportant.

C. NON-EQUILIBRIUM STATISTICAL MECHANICS

The Langevin equation model of physical processes arises in many contexts, e.g., in a geophysical context the interaction among internal waves and the interaction among surface waves linearly coupled to the turbulent wind can both be modeled by such an equation. In this model one presupposes the existence of two or more widely separated scales on which the system evolves. In a simple two-scale process, one scale characterizes the macroscopic growth time of the system while the other, which is a much shorter time scale, represents the fluctuations in the macroscopic evolution. The fluctuations provide a model of the microscopic variations in the system. The presence of fluctuations in a Langevin equation induces a spreading of the possible paths of evolution in the phase space of the system. The distribution function for these paths defines a probability density whose evolution is determined by the Langevin equation. The relation between the linear Langevin equation, i.e., an equation which is linear in the system variables and has additive fluctuations, and the equation of evolution of the probability density was developed by Fokker and Planck and now bears their names.

The phenomenon of turbulence has features in common with a great many other processes in physics. Specifically, it is again the question of the describing the dynamics of a system characterized by widely separated scales of motion (either in space, time, or both). Much of the work on turbulence can be related to topics discussed previously if we recall that homogeneous turbulence can be put in the form of a set of coupled mode rate equations with quadratic nonlinearitites. A model of turbulence suggested by Landau and studied extensively by Kraichnan supposes that the stochastic behavior of a field is introduced by means of an *ad hoc* fluctuating forcing function. This modeling of the statistical behavior of the fluid is in the spirit of Langevin's model of the motion of a Brownian particle. The properties of the solution to such a *nonlinear stochastic differential equation* are determined by all the moments of the mode amplitudes or equivalently by the probability density function describing the density of possible phase space trajectories realizable by the system.

The traditional point of view in statistical mechanics is that fluctuations, or statistics, enter into the description of deterministic systems through the process of coarse graining. This procedure reduces the number of degrees of freedom in a system from the order of 10^{23} to a few, these few generally being the conserved quantities (integral constraints) in Hamiltonian system. The degrees of freedom that have been suppressed appear as fluctuations in the equations of motion for the slowly varying quantities, e.g., the Langevin equation was constructed from precisely this argument. This coarse graining was first done to separate the effects of the truly microscopic from the macropscopic evolution of a system, e.g., the difference between the kinetic theory and the thermodynamic theory of gasses. The procedure has since been applied to almost any physical system where it is reasonable to separate time scales, e.g., the work on internal waves in B.1 above and the work on surface waves in B.3 above.

ACCOMPLISHMENTS

C.1 Resonant-Test-Field Model of Fluctuating Nonlinear Waves[51,64]

A Hamiltonian system of nonlinear dispersive waves is used as a basis for generalizing the test-wave model to a set of resonantly interacting waves. The resonant test field (RTF) is shown to obey a nonlinear generalized Langevin equation in general. In the Markov limit a Fokker-Planck equation is obtained and the exact steady-state solution is determined[51]. An algebraic expression for the power spectral density is obtained in terms of the number of resonantly interacting waves (n) in the RTF, the interaction strength (V_k), and the dimensionality of the wave field (d). For gravity waves on the ocean surface a k^{-4} spectrum is obtained, and for capillary waves a k^{-8} spectrum, both of which are in essential agreement with data[64].

This is the formal theory on which the results for the statistical properties of water waves discussed in B.3 above are dependent.

C.2 Stability Properties of Nonlinear Systems with Fluctuating Parameters[20]

Certain phenomena in the near surface ocean environment can be modeled by means of nonlinear stochastic mode rate equations. Phenomena that can be modeled in this way included nonlinear wave-wave interactions for both surface and internal waves and acoustic wave propagation in the presence of fluctuating internal waves. Aspects of the wave evolution have been studied by examining the properties of generic stochastic differential equations with fluctuating parameters, notably the linear harmonic oscillator with a fluctuating frequency, fluctuating dissipation rate and fluctuating external force. The techniques developed to solve these model systems have found immediate application in describing the initial growth phase of wind generated water waves on the ocean surface[27].

It was found that a fluctuating frequency in the linear system gives rise to moment instabilities in the state variables[2,3,7,14]. In the present study[20] we examined the stability properties of nonlinear oscillators with fluctuating potential parameters and a fluctuating additive driving force. It is shown the nonlinearities in general stabilize moments known to diverge in the corresponding linear system. This general result was implicit in obtaining the equilibrium energy spectral density in B.3.

C. 3 Macroscopic Fluid Dynamics

The studies of George Carnevale[31-32,55,56] have been related to the application of the thermodynamic principles of entropy increase to macroscopic fluid dynamics. This work neatly ties up loose ends in the Markovian Closure theory of turbulence and provides an unifying point with many-body theory. It suggests a new mode of investigation for experiments as well as introducing a new measure of predictability for the evolution of fluid flow.

The investigations are in part concerned with the interaction of waves with turbulent fluid flow. Methods of statistical turbulence theory are applied to

Rossby wave propagation and the equilibration problem. Resonant wave interaction theory, sometimes called weak interaction theory, is shown to be a limit of this more comprehensive theory and the field equations for the treatment of inhomogeneous flows are developed.

A critical analysis of viscosity renormalization schemes has been completed which suggests the appropriate method for incorporating the effects of turbulence in the field equations through the renormalization of linear parameters such as viscosity and wave frequency[55].

An exact solution for the evolving probability distribution of a two-dimensional fluid for a special form of random forcing is also obtained. The result can be of use as a test case for simulation studies as well as a limiting case for theoretical analysis[33].

D. NONLINEAR MECHANICS

The onset of stochastic behavior in dynamical systems has been the subject of intensive investigation in recent years. These dynamical systems include Hamiltonian systems of a few degrees of freedom, simple sets of difference equations (usually termed "algebraic mapping") and small sets of coupled nonlinear differential equations such as those obtained in coupling models of hydrodynamic processes. The actual prediction of a "stochastic transition" is an immensely difficult problem. A number of criteria have been proposed for Hamiltonian systems and have been recently reviewed. Although some of these methods are not suitable for the dynamical systems modeling hydrodynamic processes (the latter contain dissipative forces absent in purely Hamiltonian systems) others are worthy of further investigation. These include the method of "overlapping resonances" which has achieved great popularity in plasma physics and has the possibility of being generalized to the many types of resonant wave interactions that occur in oceanographic processes. Another possible criterion may be provided by the use of generalized Langevin equation methods. Such an approach raises interesting questions about the amount of detailed dynamical information that can be built into statistical mechanical models.

ACCOMPLISHMENTS

A number of accomplishments noted earlier also fall into this category, e.g. A.1.

D.1 Hamiltonian Techniques for Hydrodynamics

A Hamiltonian has been constructed by F. Henyey for the description of internal motions of a stratified fluid in terms of Eulerian coordinates[59]. The Hamiltonian describes both internal waves and motions with vertical vorticity and the coupling between them. The construction of the Hamiltonian is made in a way which is easily extended to any conservative non-relativistic continuim

system. This is in contrast to as hoc proceedures such as are used in the "non-canonical" approach to the same problem. Indeed, the "non-canonical" results can be derived by straight forward algebra from the canonical Hamiltonian. One key in the construction is the clarification of the role of Lin constraints.

The extension of the construction to magnetohydrodynamics was carried out[38].

In both the cases of stratified fluids and magnetohydrodynamics, the number of canonical variables exceeds the number of physical degrees of freedom, giving rise to a "gauge" invariance. The gauge groups are indentified. The group for stratified fluids is non-abelian and the group for MHD is abelian. The Lie algebra generators are conserved, leading to potential vorticity conservation in stratified fluids and to freezing in of magnetic field lines in a plasma.

The Hamiltonian form of the equations can be used for general derivation of eikonal equations[60], in which, for example, conservation of wave action is derived in a general context rather than in the case-by-case approach of previous practice.

D.2 Pseudointegrable Systems[39]

The study of chaos in dynamical systems with few degrees of freedom has become a familiar route toward understanding turbulence in fluid systems. In the limit of zero viscosity, the relevant dynamical systems are Hamiltonian.

The simple billiard is a dynamical system consisting of a mass point freely moving inside a plane, bounded region and undergoing elastic collisions with the boundaries. Studies of this system are important because they can illustrate the full range of possible behavior exhibited by continuum Hamiltonian systems. Moreover, numerical experiments are extremely easy to preform for billiard systems since the dynamics reduces to a discrete mapping between collision coordinates on the boundary. Differential equations are avoided entirely.

We have performed a detailed numerical investigation of the autocorrelation function of a billiard system which exhibits chaotic behavior. The enclosure

consists of a plane surface formed out of three squares arranged in an "L" shape. On the basis of a numerical analysis of trends of repeating "large" values of the autocorrelation function and the application of standard results in the theory of continued fractions, we were able to demonstrate that the long time behavior of the autocorrelation function is a power of law decay. From the analogy between billiard systems and continuum Hamiltonian systems we believe this behavior will be found in other continuum systems.

D.3 Analytic Structure of Dynamical Systems[10,21,36,49,58]

The study of small dynamical systems can provide useful insights into important physical processes, e.g., convective instability, molecular motion, galactic stability, wave fields, etc. Of particular interest are the circumstances under which these model systems can display chaotic (turbulent) behavior[69]. For this to occur the system has to be *nonintegrable*. Although there is no universally agreed definition of "integrability", it is generally accepted that, at least for conservative systems, it is closely related to the existence of "first integrals" of the motion. These are single-valued, analytic functions of the system coordinates and momenta that are constant along all trajectories. The more first integrals a system possesses (up to as many as there are degrees of freedom) the more restricted is the region of phase space that the trajectories can explore. Remarkably, though, there is still no standard way to determine whether a given system is, in fact, integrable. Such a determination is usually a matter of experience, luck and numerical experiment. Here we describe some new results (in fact a new application of an old idea) in this direction.

Recently members of the Center have studied the analytic structure of a variety of systems, in particular the solutions to the Henon-Heiles Hamiltonian[10,57,36]. This well-known model system consists of a pair of nonlinearly-coupled oscillators and proceedes a simple model for molecular and galactic motion. The equations are:

$$\frac{d^2x}{dt^2} = -Ax + 2Dxy$$

$$\frac{d^2y}{dt^2} = -By - Dx^2 + Cy^2 .$$

In order to find out what sort of singularities x(t) and y(t) can exhibit as a function of complex time (there are no real-time singularities for bounded motion), Tabor, Weiss, and Chang first of all perform a so-called leading-order analysis, the details of which have been published[10,36]. They demonstrate that the full solutions to equations for x(t) and y(t) can be put in the form of a Laurent series in rational powers of $(t - t_*)$ where t_* is the initial time and position of the pole.

In order to investigate the disposition of the singularities in the complex t plane further, Tabor, Weiss, and Chang made a numerical study of the differential equations[36]. This was performed using a sophisticated algorithm based on Taylor series methods that can integrate along arbitrary paths in the complex plane and locate the position and type of singularities nearest the path of integration. An astonishing structure was revealed. Around each singularity they found an equiangular, double spiral of more singularities. Each member of the spiral had its own equiangular double spiral of singularities and each member of these had its own spiral...Thus no singularity is isolated, and around each one finds a *self-similar* structure of spirals within spirals within spirals (the angle is 25°) of singularities. As one probes deeper into the complex plane the spirals become tighter and tighter, and eventually the singularities become dense at a finite distance from the real axis. This results in a *natural boundary* beyond which the solution cannot be analytically continued[57]. To date, natural boundaries tended to be discussed only in advanced mathematical texts and without any physical context. The studies of Tabor, Weiss, and Chang suggest that the self-similar boundaries are, in fact, quite common in a wide variety of physical systems. Furthermore, the actual structure of these boundaries can be determined directly from a study of the equations of motion. They have developed an asymptotic theory for the formal series expansions about a complex singularity. This theory enables a prediction of the precise structure of the spirals (usually in excellent agreement with the numerical results) and demonstrates that the spirals exist on *all scales*[50].

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